# ANALYSIS OF CHARACTERISTICS OF MIXED TRAFFIC FLOW OF AUTOPILOT VEHICLES AND MANUAL VEHICLES 

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#### Abstract

This work examines the positive effects on highway capacity implementing automatic vehicle control. Mixed traffic streams of a mimic freeway interchange simulate an on-ramp section with autopilotequipped vehicles and normal manual vehicles. The simulation illustrates characteristics of speed, volume and concentration of mixed flows. The capacity trend is presented with mixed ratios of equipped cars and its market occupation rate. In order to make current highways more efficient during the transition stage, general rules for traffic control are proposed. © 1998 Elsevier Science Ltd. All rights reserved


Keywords: highway capacity, mixed traffic stream, autopilot-equipped vehicle, Intelligent Transport Systems (ITS), Intelligent Vehicle/Highway Systems (IVHS), automatic vehicle control system (AVCS), car-following simulation.

## 1. INTRODUCTION

In Taiwan, the number of automotive owners rapidly increased over the past decade. Traffic congestion has appeared almost everywhere at the same time in both urban areas and on freeways. Because of the lack of land resources and low cost effectiveness, increased road construction is not a complete solution to the problem. Using advanced technologies to improve highway capacity provides an effective solution. Intelligent Vehicle/Highway Systems (IVHS) or Intelligent Transport Systems (ITS) are a global trend in transportation research. ITS attempts to overcome traffic bottlenecks and blind spots to achieve transportation efficiency and safety. An automatic vehicle control system (AVCS) within ITS emphasizes enhanced highway capability by using electronic, communicative and computerized equipment in vehicles. Many institutes and automotive industries have extensively addressed this issue (see Nwagboso, 1993; IEEE, 1994; SAEJ, 1994; ITS America, 1995, 1996; VERTIS, 1995). Road and Vehicle Automation is a more practical solution.
An Automatic Vehicle Control System named ADVANCE-F has been developed in Taiwan since 1990 . This project aims to upgrade highway capacity and traffic safety by applying advanced techniques to control moving vehicles (Chang, 1992). Intelligent vehicle control subsystems have been tested. From preliminary results of these tests, this system can hopefully be implemented in the near future. Figure 1 describes a system for the ADVANCE-F vehicle. The equipped car has functions of automatic lateral and longitudinal control. Steering of the equipped car can be manual or automatic. Its speed control has three options: manual driving, conventional cruise keys, and autonomous intelligent cruise control. Our previous work proposed a central-hazardavoidance remote system for a subsequent project. (Chang, 1993, 1994).

Before product marketing, traffic engineers must understand as accurately as possible the traffic characteristics of mixed flows of equipped and non-equipped vehicles. Owing to this reason, traffic

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Fig. 1. Scheme of an ADVANCE-F vehicle.
management must be considered. Consequently, this paper analyzes traffic characteristics such as the relationship between volume, speed, and concentration (density) under various composition ratios of vehicle handling types. Above simulation results in time and space are based on the carfollowing policy initiated in ADVANCE-F. In simulation, except diverging behavior, merging and following are described. Every moving phenomenon is tracked and monitored with time-space diagrams. A tendency of highway capacity is expressed in terms of the rate of market occupation of equipped cars. Finally, an overall strategy is proposed to control traffic during a transition period during implementation, particularly in operating and managing mixed flow patterns. The effectiveness of an Automated Highway System (AHS) is proved then.

Many references simulate and discuss flow benefits of AVCS as well as AHS (Agre and Clare, 1993; Rao and Varaiya, 1993; Rao, Varaiya and Eskafi, 1993). However, different systems have different assumptions and scenarios. Moreover, different areas like Taiwan and Asia have different traffic behaviors. This work describes the model and the decision making of car-following and merging before simulation.

## 2. CAR-FOLLOWING ALGORITHM OF ADVANCE-F VEHICLES

An understanding of an ADVANCE-F vehicle must precede the simulation.

### 2.1. Minimum intervehicle spacing

When an equipped car is maneuvered with the automatic control mode of the ADVANCE-F system, it must follow a vehicle in a lane, with a target for capacity. Also a minimum spacing for safety is needed. Basically, the most critical situation to decide the minimum spacing of cars is when its preceding vehicle suddenly brakes. To avoid hitting its preceding vehicle, four terms are considered in the minimum spacing calculation. These terms that determine the travel distance during sensing, decision and actuating delay, the travel distance in the time from initiating brake to achieving maximum brake, the travel distance while stopping, and the distance traveled by the preceding vehicle with maximum deceleration. The four terms are sequentially presented at right-hand-side of eqn (1) (Fenton, 1979):

$$
\begin{equation*}
S_{\mathrm{opt}}=V_{0} \tau+\left(V_{0} \frac{D_{\max }}{J_{\max }}-\frac{1}{6} \frac{D_{\max }^{3}}{J_{\max }^{2}}\right)+\frac{\left(V_{0}-\frac{D_{\max }^{2}}{2 J_{\max }}\right)^{2}}{2 D_{\max }}-\frac{V_{1}^{2}}{2 D_{\max }^{\prime}} \tag{1}
\end{equation*}
$$

where $V_{0}$ denotes the velocity of the equipped vehicle ( $\mathrm{m} \mathrm{s}^{-1}$ ) of interest, $\tau$ represents the total mechanical delay (s) including sensing delay of the equipment, decision delay of the controller and braking delay, $D_{\max }$ is the maximum deceleration ( $\mathrm{m} \mathrm{s}^{-2}$ ), $J_{\max }$ denotes the maximum jerk rate $\left(\mathrm{m} \mathrm{s}^{-3}\right)$, and $V_{1}$ and $D_{\text {max }}^{\prime}$ represent the velocity and the maximum deceleration capability of the preceding vehicle, respectively. For any equipped vehicle in fair climate, parameter $\tau, D_{\max }$ and $J_{\text {max }}$ are known to be $0.245 \mathrm{~s}, 7.85 \mathrm{~m} \mathrm{~s}^{-2}$ and $76.2 \mathrm{~m} \mathrm{~s}^{-3}$. Thus, $S_{\mathrm{opt}}$ in eqn (1) is less than and approximate to

$$
\begin{equation*}
\left(\tau+\frac{D_{\max }}{J_{\max }}\right) V_{0}+\frac{V_{0}^{2}}{2 D_{\max }}-\frac{V_{1}^{2}}{2 D_{\max }^{\prime}} \tag{2}
\end{equation*}
$$

According to the system function of ADVANCE-F, with a camera mounted behind the front windshield and a specific tag placed on the rear bumper, each equipped car can recognize if its preceding car is equipped or non-equipped by image processing (Chang, 1993, 1994). Also, each equipped car can measure the speed $V_{1}$ and the deceleration of the preceding car through the mounted laser beam range finder. If the preceding car is equipped, based on eqn (2), the minimum following spacing for an equipped vehicle is

$$
\begin{equation*}
S_{\mathrm{rel}}=0.35 V_{0}+0.0637\left(V_{0}^{2}-V_{1}^{2}\right) \tag{3}
\end{equation*}
$$

However, if its preceding car is non-equipped, the action of the preceding car is unpredictable. This unpredictability may put psychological pressure on the driver as well as the passengers in the equipped vehicle because of the short following distance of $S_{\text {rel }}$. An arbitrary vehicle following situation, like handling a non-equipped vehicle, is considered for the equipped car. According to the California rule, for tight following every vehicle should keep the spacing of one vehicle length for every 10 mph in order to have enough time for response. The spacing is $0.225 L_{\mathrm{V}} \mathrm{V}_{0}$ (Ioannou et al., 1992), where $L_{\mathrm{v}}$ is the vehicle length in meters and $V_{0}$ is the speed in $\mathrm{m} \mathrm{s}^{-1}$. Taiwan also has the same rule for driving. Thus, if its preceding car is non-equipped, the minimum following spacing for an equipped vehicle is set to be

$$
\begin{equation*}
S_{\mathrm{abs}}=0.225 L_{\mathrm{v}} V_{0}+0.0637\left(V_{0}^{2}-V_{1}^{2}\right) \tag{4}
\end{equation*}
$$

Considering the mean of vehicle lengths in Taiwan, eqn (4) implies that

$$
\begin{equation*}
S_{\mathrm{abs}}=1.0125 V_{0}+0.0637\left(V_{0}^{2}-V_{1}^{2}\right) \tag{5}
\end{equation*}
$$

### 2.2. Principles of following speed

The following speed of the equipped car is determined according to the following rules. If the preceding car is equipped and has a speed of $V_{1}, \quad V_{0} \rightarrow V_{1}$ is assigned if their spacing $S$ satisfies $S_{\text {rel }} \leq S \leq S_{\text {rel }}+1.5$, where 1.5 is the defined allowance in meters.

If $S>S_{\text {rel }}+1.5$, the equipped follower is allowed to accelerate until $S_{\text {rel }} \leq S \leq S_{\text {rel }}+1.5$ is reached. Section 2.4 describes the acceleration rate. During acceleration, $V_{0}$ should not exceed $33.4 \mathrm{~m} \mathrm{~s}^{-1}$ because the speed limit is predetermined for the equipped car at 120 kph . For some situations, $S<S_{\text {rel }}$ and $V_{0}>V_{1}$ simultaneously, the equipped follower should decelerate until the condition, $S_{\text {rel }} \leq S \leq S_{\text {rel }}+1.5$, is satisfied. If $S<S_{\text {rel }}$ but $V_{0}<V_{1}$, the equipped follower should wait for a little time before accelerating to constitute the minimum following spacing rule, i.e. $S=S_{\text {rel }}$ and $V_{0}=V_{1}$. Providing that the follower will take $t$ seconds with the acceleration rate $A_{\mathrm{n}}$ to reach $V_{0}=V_{1}$, or said $V_{1}=V_{0}+A_{\mathrm{n}} t$, this implies $t=\left(V_{1}-V_{0}\right) / A_{\mathrm{n}}$. Therefore, while $S<S_{\text {rel }}$ and $V_{0}<V_{1}$, the situation of the equipped follower beginning to accelerate is set at

$$
\begin{equation*}
S=S_{\text {rel }}-V_{1} t+\frac{1}{2} A_{\mathrm{n}} t^{2}=S_{\text {rel }}-\frac{1}{A_{\mathrm{n}}}\left[V_{1}\left(V_{1}-V_{0}\right)-\frac{1}{2}\left(V_{1}-V_{0}\right)^{2}\right]=S_{\text {rel }}-\frac{1}{2 A_{\mathrm{n}}\left(V_{1}^{2}-V_{0}^{2}\right)} \tag{6}
\end{equation*}
$$

For a comfortable ride, $A_{\mathrm{n}}$ is set to be less than $0.2 \boldsymbol{g}\left(1.95 \mathrm{~m} \mathrm{~s}^{-2}\right)$. (Matsui, 1962; Fenton and Chu, 1977; Hauksdottir and Fenton, 1986).
If its preceding car is non-equipped and has speed $V_{1}, V_{0} \rightarrow V_{1}$ is assigned if their spacing $S$ satisfies both $S_{\text {abs }} \leq S \leq S_{\text {abs }}+1.5$ and $V_{1} \leq 33.4 \mathrm{~m} \mathrm{~s}^{-1}$, however, $V_{0}=33.4$ if $V_{1}>33.4 \mathrm{~m} \mathrm{~s}^{-1}$. If $S>S_{\text {abs }}+1.5$, the equipped follower is allowed to accelerate until $S_{\text {abs }} \leq S \leq S_{\text {abs }}+1.5$ holds; however $V_{0}$ should be less than $33.4 \mathrm{~m} \mathrm{~s}^{-1}$. If $S<S_{\text {abs }}$ and $V_{0}>V_{1}$, the equipped follower should decelerate until the condition, $S_{\mathrm{abs}} \leq S \leq S_{\mathrm{abs}}+1.5$, is satisfied; however, if $V_{0}<V_{1}$ and $V_{1} \leq 33.4 \mathrm{~m} \mathrm{~s}^{-1}$, the spacing becomes

$$
\begin{equation*}
S=S_{\mathrm{abs}}-\frac{1}{2 A_{\mathrm{n}}}\left(V_{1}^{2}-V_{0}^{2}\right) \tag{7}
\end{equation*}
$$

Then, the equipped follower accelerates such that $V_{0}=V_{1}$. In the same spacing situation, if $V_{0}<V_{1}$ but $V_{1}>33.4$, then $V_{0}=33.4$. Similar as before, $A_{n}$ is set to be less than $0.2 \boldsymbol{g}\left(1.95 \mathrm{~m} \mathrm{~s}^{-2}\right)$.

### 2.3. Estimation of the speed, acceleration and deceleration of the preceding car

By using a laser-beam range finder (Fig. 1), an equipped car estimates the speed and acceleration or deceleration of its preceding car from time to time by measuring the intervening distance. However, for the computer simulation in the following section, a simplified model is substituted for the function of the laser-beam range finder. Assume that $S_{\mathrm{t}}$ is an intervehicle spacing measured by an equipped follower at time $t$. Thus,

$$
\begin{equation*}
\Delta S=S_{\mathrm{t}}-S_{\mathrm{t}-1} \tag{8}
\end{equation*}
$$

expresses the spacing variation in a scanning interval $\Delta t$. Hence,

$$
\begin{equation*}
V_{1}=V_{0}+\Delta S / \Delta t \tag{9}
\end{equation*}
$$

The difference between acceleration of two cars is estimated according to

$$
\begin{equation*}
\Delta A=A_{1}-A_{0}=\frac{1}{2} \frac{\Delta S}{\Delta t^{2}} \tag{10}
\end{equation*}
$$

or

$$
\begin{equation*}
A_{1}=A_{0}+\frac{1}{2} \frac{\Delta S}{\Delta t^{2}} \tag{11}
\end{equation*}
$$

in which $A_{1}$ and $A_{0}$ are denoted as the acceleration or deceleration of the preceding vehicle and the equipped follower, respectively.

Consequently, micro-adjusting the velocity of the equipped follower relies on $\Delta A$ described above. However, the comfortable range of acceleration and deceleration is restricted to the range of $-0.2 \boldsymbol{g} \sim 0.2 \boldsymbol{g}$. Indeed, as described above, eqns (9) and (11) are only utilized for the traffic simulation in this study. They are not really applied for the speed estimation and longitudinal control in the ADVANCE-F system. In the real vehicle control program, to overcome the reduced signal to noise ratio, there is a Kalman Filter designed to manipulate the estimation, depending on the time series $\{\Delta S\}$.

### 2.4. Decision of acceleration or deceleration

According to the above scheme, the whole operation relies on the acceleration or deceleration value of equipped cars $A_{\mathrm{n}}$ (i.e. $A_{0}$ in Section 2.3). With an optimal control model, the acceleration or deceleration value is calculated. Based on dynamics, it holds (Chang, 1994)

$$
\frac{\mathrm{d}}{\mathrm{~d} t}\left[\begin{array}{c}
\dot{x}_{1} \\
x_{1} \\
\dot{x}_{0} \\
x_{0}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{array}\right]\left[\begin{array}{c}
\dot{x}_{1} \\
x_{1} \\
\dot{x}_{0} \\
x_{0}
\end{array}\right]+\left[\begin{array}{l}
0 \\
0 \\
1 \\
0
\end{array}\right]\left[\ddot{x}_{0}\right]
$$

or briefly

$$
\begin{equation*}
\dot{\mathbf{X}}=\mathbf{A X}+\mathbf{B u} \tag{12}
\end{equation*}
$$

where $x_{0}, x_{1}$ denote the travel distance of the equipped follower and its preceding car; $\dot{x}_{0}$ and $\dot{x}_{1}$ represent the velocities of the equipped follower and its preceding car, respectively. $\mathbf{u}=\left[\ddot{x}_{0}\right]$ is the acceleration or deceleration of the equipped car.
$S$ denotes the real spacing between the preceding car and the equipped follower. Thus, for the consideration of safety and capacity, it is anticipated that, $S+x_{1}-x_{0}=d_{\mathrm{s}}$ or

$$
\begin{equation*}
S-d_{\mathrm{s}}=x_{0}-x_{1} \tag{13}
\end{equation*}
$$

where $d_{\mathrm{s}}$ could be $S_{\mathrm{rel}}$ or $\mathrm{S}_{\mathrm{abs}}$ dependent on if the preceding car is an equipped or non-equipped vehicle. Then,

$$
\mathbf{E}=\left[\frac{\dot{x}_{0}-\dot{x}_{1}}{S-d_{\mathrm{s}}}\right]=\left[\begin{array}{cccc}
-1 & 0 & 1 & 0  \tag{14}\\
0 & -1 & 0 & 1
\end{array}\right] \mathbf{X}
$$

By considering eqns (12) and (14), the following equation is satisfied:

$$
\dot{\mathbf{E}}=\left[\begin{array}{ll}
0 & 0 \\
1 & 0
\end{array}\right] \mathbf{E}+\left[\begin{array}{l}
1 \\
0
\end{array}\right] \mathbf{u}
$$

or in brief

$$
\begin{equation*}
\dot{\mathbf{E}}=\tilde{\mathbf{A}} \mathbf{E}+\tilde{\mathbf{B}} \mathbf{u} \tag{15}
\end{equation*}
$$

The feedback loop of the dynamic system is defined as

$$
\begin{equation*}
\mathbf{u}=-\mathbf{k} \cdot \mathbf{E} \tag{16}
\end{equation*}
$$

By minimizing the performance function

$$
\begin{equation*}
J=\frac{1}{2} \int_{0}^{\infty}\left(\mathbf{E}^{\mathrm{T}} \mathbf{Q} \mathbf{E}+\mathbf{u}^{\mathrm{T}} \mathbf{R} \mathbf{u}\right) \mathrm{d} t \tag{17}
\end{equation*}
$$

the feedback gain $\mathbf{k}$ is

$$
\begin{equation*}
\mathbf{k}=\mathbf{R}^{-1} \tilde{\mathbf{B}}^{\mathrm{T}} \Pi_{\infty} \tag{18}
\end{equation*}
$$

where $\Pi_{\infty}$ denotes a $\Pi$ satisfying the Riccati equation with infinite termination (Kuo, 1991):

$$
\begin{equation*}
\Pi \tilde{\mathbf{A}}+\tilde{\mathbf{A}}^{\mathrm{T}} \Pi-\Pi \tilde{\mathbf{B}} \mathbf{R}^{-1} \tilde{\mathbf{B}}^{\mathrm{T}} \Pi \mathbf{Q}=0 \tag{19}
\end{equation*}
$$

In eqn (17), the weighting matrices

$$
\mathbf{Q}=\left[\begin{array}{cc}
q_{1} & 0 \\
0 & q_{2}
\end{array}\right], \quad \mathbf{R}=[r]
$$

A general rule to select the weighting factors is to take the reciprocal of the square of the allowed or maximum value of the relative variables so as to normalize each term in the performance function. Since

$$
\left(\dot{x}_{0}-\dot{x}_{1}\right)^{2} \leq 2 D_{\max } \Delta x
$$

$\Delta x=\left|x_{0}-x_{1}\right|$, where

$$
\begin{gathered}
q_{1}=1 /\left(2 D_{\max } \Delta x\right) \\
q_{2}=1 / \Delta x^{2} \\
r=1 / D_{\max }^{2}
\end{gathered}
$$

By eqn (13), $\Delta x$ can be substituted for $\left|S-d_{\mathrm{s}}\right|$. Where $D_{\max }$ equals $\mu \boldsymbol{g}$ (Vos, 1992). Also, $\mu$ represents the coefficient of longitudinal friction between tires and pavement and $\boldsymbol{g}$ denotes the gravitational acceleration. $\mu=0.4 \sim 0.8$ dependent upon the road conditions. However, $D_{\max }=7.85$ corresponds to $\mu=0.8$ is considered to find the maximum capacity.
By calculating eqn (19) and substituting the weighting factors, the feedback gain $\mathbf{k}$ in eqn (18) yields

$$
\mathbf{k}=\left[\begin{array}{ll}
\sqrt{\frac{1}{r}\left[q_{1}+2\left(r q_{2}\right)^{\frac{1}{2}}\right.} & \left.\sqrt{\frac{1}{r} q_{2}}\right]=\left[\begin{array}{ll}
\sqrt{\frac{5 \mu g}{2\left|S-d_{s}\right|}} & \left.\sqrt{\frac{\mu g}{\mid S-d_{s}}}\right]
\end{array}\right] . \tag{20}
\end{array}\right.
$$

Thus, by eqns (14) and (20), the time-dependent eqn (16), i.e. the acceleration or deceleration of the equipped car, is

$$
\mathbf{u}=\left[\ddot{x}_{0}\right]=-\left[\begin{array}{ll}
\frac{5 \mu g}{2\left|S-d_{\mathrm{s}}\right|} & \left.\frac{\mu g}{\mid S-d_{\mathrm{s}}}\right]
\end{array}\right]\left[\begin{array}{l}
\dot{x}-\dot{x}  \tag{21}\\
S-d_{\mathrm{s}}
\end{array}\right]=\sqrt{\frac{d D_{\text {max }}}{2 \mid S-d_{\mathrm{s}}}}\left(V_{1}-V_{0}\right)+\frac{D_{\text {max }}}{\left|S-d_{\mathrm{s}}\right|}\left(d_{\mathrm{s}}-S\right)
$$

Equation (21) is utilized to adjust the acceleration of the equipped vehicles in the following simulation. Nevertheless, the acceleration or deceleration is set to be less than $0.2 \mathrm{~g}\left(1.95 \mathrm{~m} \mathrm{~s}^{-2}\right)$, as described in Section 2.2

## 3. SIMULATION MODELING

Freeway capacity is normally determined with the factors of facility operations and traffic characteristics around interchanges. Therefore, analyzing the capacity of mixed flows in freeway traffic should be carried out on sections close to interchanges. Because weaving and diverging are similar to merging behavior in each analytical action, the traffic situation is simplified so that only merging and following events occur in the simulation. The simulation emphasizes the occasion of down stream converging of the on-ramp. Only merging and following are considered, assuming no weaving and diverging events.

### 3.1. Environment of the simulated road

In accordance with the above simplification, the flow was simulated and observed at a 4 km section of a mimicked motorway consisting chiefly of a major lane and an on-ramp acceleration lane. Figure 2 illustrates the layout of this junction section. The length of the acceleration lane is 270 m and is coupled with the major lane which begins 1 km away from the point of generating through traffic. The road is assumed to be flat and straight. For convenience in simulating the pointer setting, the co-ordinates of the acceleration lane is counted from $1^{\mathrm{k}}+000$ to $1^{\mathrm{k}}+270$.

### 3.2. Traffic generation

At the beginning of the mimicked lanes, a major lane and an acceleration lane, vehicles are generated by an arrival function. The arrival function is a shifted negative exponential distribution with a shifted parameter of 0.3 . The 'shifted' represents a prohibition of initial headways less than 0.3 s (Gerlough and Huber, 1975). Each vehicle is labeled with the attributes equipped or nonequipped, length, width, capability of maximum acceleration, capability of maximum deceleration,


Fig. 2. The layout of a mimicked junction section.
time of arrival, identification code, expected speed, current lane occupied, moving status (following, merging or desired), time of entering the major lane, location of entering the major lane, current speed, current location, pointer of accelerative or decelerative levels, and front spacing of the last scanning. Vehicle handling with automatic or manual means is distributed according to a vehicle generation function initiated with a given fraction. Length, width, maximum acceleration and maximum deceleration are considered for 20 kinds of cars that are most common in Taiwan traffic. The expected speed of equipped vehicles in an ideal environment is $33.3 \mathrm{~m} \mathrm{~s}^{-1}(120 \mathrm{kph})$. Most traffic studies have demonstrated that free-flow speeds constitute normal distributions (Gerlough and Huber, 1975; Homburger, 1982). In Taiwan, freeway speed was also verified to be normal (Chang, Huang and Kerr, 1989). However, different sections have different concentrations and deviations. In simulating non-equipped cars the expected speed for free flow is given by a normal distribution with mean $30.5 \mathrm{~m} \mathrm{~s}^{-1}$ and standard deviation $2.8 \mathrm{~m} \mathrm{~s}^{-1}$. Because the car generating points are located upstream from an interchange or on-ramp, diverging and weaving traffic slow the speed down. Thus, the prevailing speed $V\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ for a vehicle, is initially assigned by the normal distributions: $\mathrm{N}\left(21.5,3.5^{2}\right)$ and $\mathrm{N}\left(16.0,3.5^{2}\right)$, respectively for the major lane and the acceleration lane. The data reveals that the speed output is confined within the interval of $72 \sim 97 \mathrm{kph}$ if vehicles are generated from the major lane and is bounded by $45 \sim 70 \mathrm{kph}$ for the acceleration lane. During simulation, the latter eight attributes are updated in each scan.

### 3.3. Following basic

The car-following model for equipped vehicles is described in Section 2. Hereafter in this section, a model for non-equipped vehicles is presumed. If there is no effective interference preceding a non-equipped vehicle, the vehicle runs with or accelerates approaching its expected speed (initiated by $\mathrm{N}\left(30.5,2.8^{2}\right)$. If a car is in front of the simulated car, minimum spacing should be maintained for safety. According to the analysis of Chang et al. (1989) Greenshields' linear speedconcentration model (Greenshields, 1934) is sufficient to explain Taiwan's traffic phenomena. Based on the Greenshields' model and the conducted survey, the minimum car-following spacing is investigated as

$$
\begin{equation*}
S_{\text {app }}=0.20995 \cdot \mathrm{~V}+6.4=0.20995 \cdot 3.6 \mathrm{~V}+6.4 \tag{22}
\end{equation*}
$$

in which $\mathbf{V}$ denotes the speed in kilometers per hour (kph) and $V$ is expressed in $\mathrm{m} \mathrm{s}^{-1}$.
Equation (22) is applied for checking car-following limit of non-equipped vehicles in our simulation. Furthermore, the derivative of eqn (22) is, the acceleration or deceleration law

$$
\frac{\mathrm{d}}{\mathrm{~d} t} S_{\mathrm{app}}=0.75582 \frac{\mathrm{~d}}{\mathrm{~d} t} V
$$

implies

$$
\begin{equation*}
A_{0}=1.323\left(V_{1}-V_{0}\right) \tag{23}
\end{equation*}
$$

Equation (23) is utilized to accelerate or decelerate non-equipped vehicles in simulation. Nevertheless, under normal conditions, acceleration or deceleration is maintained within the interval $(-0.2 g, 0.2 g)$ and, in each 0.3 s scan, the largest variation of acceleration is confined to one fifth of the capable maximum acceleration.

### 3.4. Merge model

According to Fig. 2, vehicles from the acceleration lane merge only left into the stream of the major lane.

1. Gap Acceptance of Equipped Vehicles: when the left-turn signal is on, the acceptable merging gap $G$ is detected with infrared sensors mounted on the left-front and left-rear sides of the equipped car. $G$ is the spacing that satisfies $G \geq 2 S_{\text {rel }}+L_{\mathrm{v}}$ if the spacing is between two equipped cars. $G \geq S_{\text {abs }}+L_{\mathrm{v}}+S_{\text {rel }}$ if the spacing is between a non-equipped vehicle and an equipped car. $G \geq S_{\mathrm{abs}}+L_{\mathrm{v}}+S_{\text {ava }}$ if the spacing is between two non-equipped cars. $L_{\mathrm{v}}$ is the length of the cars, and $S_{\mathrm{ava}}=0.36 \cdot L_{\mathrm{v}} \cdot V_{\mathrm{r}} ; V_{\mathrm{r}}$ is the velocity of the trailing car. $S_{\mathrm{ava}}$ is used instead of $S_{\text {app }}$ if the gap acceptance decision is based on safety consideration on account of the Taiwan driver response time-PIEV (Perception, Identification, Judgment and Reaction) when a vehicle merges into the lane. $S_{\text {ava }}$ is more conservative than $S_{\text {app }}$ in eqn (22).
2. Gap Acceptance of Non-Equipped Vehicles: as non-equipped vehicles lack advanced support, merging depends on a driver's judgment. According to Wang's study (1986), a lanechange requires at least 2 s , but over 4 s is more appropriate. However, a forced merging usually takes between 2 and 4 s . Since the length of acceleration lane on ramp has a limit, forced merging frequently occurs in Taiwan traffic. Thus, in this simulation, the gap $G \geq t_{\mathrm{m}} \cdot V_{\mathrm{m}}, t_{\mathrm{m}}=2$, is acceptable for merging of a non-equipped vehicle. $V_{\mathrm{m}}$ denotes the approaching speed along the road when merging. Since the merging vehicles synchronize longitudinal and lateral movement from a lane to another lane, lateral displacement affects the calculation of longitudinal or approaching velocity. Supposing the roadway is straight, the approaching speed is calculated with the form

$$
\begin{equation*}
V_{\mathrm{m}}=\sqrt{V_{0}^{2}-\left(w / t_{\mathrm{m}}\right)^{2}} \tag{24}
\end{equation*}
$$

$w$, the width of the lane, is normally 3.75 m . The term $w / t_{\mathrm{m}}$ is a lateral speed. $V_{0}$ is the heading speed.
3. Decision of Merging: when a car is generated in the mimicked acceleration lane, regardless of whether it is equipped or non-equipped, it begins to find an acceptable gap to merge into the left major lane. The decision loop is presented as following:
(i) If a current gap is acceptable, the vehicle begins to merge. However, if the gap is sufficient at this moment, immediate merging may cause the merging car to close on the preceding vehicle. This violates the minimum following spacing and interrupts the trailing vehicle's minimum spacing requirement. Step (ii) is triggered to avoid a rear-end collision. Step (iii) is intended to elude serious interruption of the trailing vehicle to avoid causing a slinky effect and a chaining rear-end collision.
(ii) The merge-desirable vehicle in the acceleration lane should slightly decelerate until the spacing $H$ is appropriate. The possible merging point to the preceding car must satisfy the following conditions and then merges into the major lane. If both the merge-desirable car and the preceding car are equipped, in order to meet the minimum following spacing (described in Section 2.1) while merge is complete, $H$ is defined as

$$
\begin{equation*}
H=S_{\mathrm{rel}}-t_{\mathrm{m}}\left(V_{1}-V_{\mathrm{m}}\right) \tag{25}
\end{equation*}
$$

The second term of the right-hand side of eqn (25) is exactly a variation of spacing during merge. After merge, the car turns to follow the car-following rule described in Section 2. If the merge-desirable car is equipped, but the preceding car is non-equipped, similarly

$$
\begin{equation*}
H=S_{\mathrm{abs}}-t_{\mathrm{m}}\left(V_{1}-V_{\mathrm{m}}\right) \tag{26}
\end{equation*}
$$

$t_{m}=2$ can be applied to eqns (25) and (26) during rush traffic.
If the merge-desirable car is non-equipped, the preceding car equipped or non-equipped will be neglected in the decision, because the merge-desirable car cannot identify what it is. By assuming that the gap is merely equal to the minimum acceptable gap for non-equipped vehicles (see 2. in Section 3.4), i.e. $G=t_{\mathrm{m}} \cdot V_{\mathrm{m}}$, the merge occurs most probably at the middle point of the length $t_{\mathrm{m}} V_{\mathrm{m}}$, i.e. at $t_{\mathrm{m}} V_{\mathrm{m}} / 2$. If the merge is a forced merging, it will cause the trailing car to decelerate; thereafter the drivers will adjust the following spacings themselves. If the gap is greater than the minimum acceptable gap, i.e. $G>t_{\mathrm{m}} \cdot V_{\mathrm{m}}$, the merging point is a random variable outcoming from a certain interval $Z$ within the acceptable gap $G$ without $t_{\mathrm{m}} \mathrm{V}_{\mathrm{m}}$. That is $Z=G-t_{\mathrm{m}} V_{m}$. However, $Z$ is meaningless if $Z$ is too large on the limiting acceleration lane as well as decelerating to find an appropriate location to merge. In our observation, $99 \%$ of the $Z \mathrm{~s}$ are less than $4 L_{\mathrm{v}}$. For convenience, $Z=\min \left\{G-t_{\mathrm{m}} V_{\mathrm{m}}, 4 L_{\mathrm{v}}\right\}$ is chosen as the domain of the random variable merging point in the simulation (Chang, 1993).

The merge point is measured a distance $X Z$ away from the origin of $Z$ (backward to the preceding car), where $X$ is a random variable between 0 and 1 . Therefore, the possible spacing between the merging car and its preceding vehicle is estimated as

$$
\begin{equation*}
H=\frac{t_{\mathrm{m}} \cdot V_{\mathrm{m}}}{2}+X Z \tag{27}
\end{equation*}
$$

Since $X$ is a random variable with respect to the merging point and $0 \leq X \leq 1$, and having most possibility for the merge almost closing to the origin of $Z$ in the problem of decelerating to find an appropriate $H$ to merge, $X$ is presumed and tested to constitute the Beta probability distribution:

$$
\begin{equation*}
\int_{0}^{X} \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1} d x=\zeta \tag{28}
\end{equation*}
$$

where $\alpha=3$ and $\beta=5$ (Chang, 1993). The purpose of $X$ in simulation is to obtain a random number from 0 to 1 for $\zeta$ and then a numerical transformation is made with eqn (28). For more about Beta distributions, the books of Roussas (1997), Mendenhall and Scheaffer (1973) are referred.
(iii) The merge-desirable vehicle accelerates forward to a possible merging point which is located at $H^{\prime}$ in front of the trailing car on the major lane and merges into the major lane. According to the concept described in (ii), if both the merge-desirable car and the trailing car are equipped, $H^{\prime}$ is defined as:

$$
\begin{equation*}
H^{\prime}=S_{\mathrm{rel}}-t_{\mathrm{m}}\left(V_{\mathrm{m}}-V_{\mathrm{r}}\right) \tag{29}
\end{equation*}
$$

If the merge-desirable car is equipped, but the trailing car is non-equipped, then

$$
\begin{equation*}
H^{\prime}=S_{\mathrm{app}}-t_{\mathrm{m}}\left(V_{\mathrm{m}}-V_{\mathrm{r}}\right) \tag{30}
\end{equation*}
$$

If the merge-desirable car is non-equipped, despite what the trailing car is,

$$
\begin{equation*}
H^{\prime}=\frac{t_{\mathrm{m}} \cdot V_{\mathrm{m}}}{2}+(1-X) Z \tag{31}
\end{equation*}
$$

in which $X$ and $Z$ are similarly defined in (ii); however $\alpha=5$ and $\beta=3$ (Chang, 1993).
(iv) If the present gap left of the merge-desirable car is inappropriate, there are two possible options. First, an appropriate gap must exist for an acceleration of $t^{\prime} \mathrm{s}$. The threshold of $t^{\prime}$ is found to be 11 s , i.e. $t^{\prime}<11$ (Chang et al., 1989; Lai, 1994) and the possible forwarding distance is constrained by the acceleration lane. If there is an appropriate gap for a merge and the remainder of the acceleration lane suffices for overtaking, then (v) is executed; otherwise, (vi) is performed.
(v) (v) The merge-desirable vehicle quickly accelerates forward, overtaking cars, to the nearest merge-possible point. When the merge-desirable vehicle reaches the merging point, $H^{\prime}$ in front of the car just passed, it then merges. The decision is similar to step (iii).
(vi) The merge-desirable vehicle in the acceleration lane should slightly decelerate until the spacing is acceptable. The merging point should be in an acceptable gap and $H$ long behind the car just passed with the accepted gap $G$. This way is like step (ii).
(vii) If the merge-desirable vehicle arrives at the end of the acceleration lane and is still in a gap-searching state, it has to stop. At this time, a queue forms.


Fig. 3. Vehicle traces on the major lane.

## 4. VALIDATION AND RESULT ANALYSIS

A time-scanning method is applied in this simulation. The scanning cycle is set to 0.3 s . Before accepting the result, the simulation was validated by the Chi-square tests with a level of significance of $5 \%$. The tests include random numbers, arrival distributions, speed distributions, merging distributions, and headway distributions. In addition, examining and explaining the vehicle traces in the simulation outputs is important to understanding the behavior of vehicles. Figure 3(a) and (b) shows partial vehicle traces on the major lane. Clearly a vehicle's behavior affects its trailing cars' behavior. Also, from Fig. 3(b), the shock has transmission delay. Such a phenomenon occurs in the real world. Fig. 4(a) illustrates vehicle traces during the stage of merging. In the figure, 'vehicle $x x$ in line $l$ ' expresses the vehicle xx is in the acceleration lane, 'vehicle $x x$ in line 2' says that vehicle xx is in the major lane, and 'vehicle $x x$ in line 0 ' expresses the vehicle is merging, simultaneously occupying the acceleration lane and the major lane. Figure 4(a) shows the change of vehicle status of lane use. Figure 4(b) reveals that there is a car accelerating and passing another car before merging.


Fig. 4. Vehicle traces during merging.


Fig. 5. V-Q diagram (a) mixed ratio 0.0 ; (b) mixed ratio 0.3 ; (c) mixed ratio 0.5 ; (d) mixed ratio 0.8 ; (e) mixed ratio 1.0 .

The model has been proven valid. Various scenarios were simulated to demonstrate the characteristics of traffic. The simulation results are presented herein for given varied arrival rates in the major and acceleration lanes with a given fraction of equipped vehicles mixed in with flow. Figure 5 shows the relationship of volume and velocity ( $\mathrm{Q}-\mathrm{V}$ ) with a varied mixture of autopilot vehicles. Based on values of maximum flow, the roadway capacity increases as the ratio of equipped vehicles grows. However, the increment is more significant if the ratio exceeds 0.5 . When all cars are equipped, the capacity increases about $33 \%$.


Fig. 6. $\mathrm{K}-\mathrm{V}$ diagram (a) mixed ratio 0.0 ; (b) mixed ratio 0.3 ; (c) mixed ratio 0.5 ; (d) mixed ratio 0.8 ; (e) mixed ratio 1.0 .

Figure 6 plots the inclination of velocity vs density (V-K) analyzed from the simulations. Velocity decreases abruptly when density lies between 20 and 40 vehicles $\mathrm{km}^{-1}$. Clearly the slope of the V-K curve becomes negatively large with increasing equipped vehicles. Hence equipped cars are more sensitive to driving circumstances. Figure 7 illustrates the variation of the $\mathrm{Q}-\mathrm{K}$ relationship. When more equipped cars are mixed into the flow, the traffic stream is better, because equipped cars in a platoon can be organized and have stable speed and spacing unlike nonequipped vehicles exposed to large variation of both speed and spacing.


Fig. 7. Q-K diagram (a) mixed ratio 0.0 ; (b) mixed ratio 0.3 ; (c) mixed ratio 0.5 ; (d) mixed ratio 0.8 ; (e) mixed ratio 1.0 .

From a practical point of view, the capacity of a highway is really determined by its operation at intersections. The critical section of a freeway's capacity is at interchanges. In accordance with our simulation results, the capacity of a freeway is fitted as:

$$
\begin{equation*}
Q_{\max }=2075.95+592.99 R-1312.84 R^{2}+1386 R^{3} \tag{32}
\end{equation*}
$$

and illustrated in Fig. 8, where $R$ denotes the ratio of equipped vehicles in the traffic flow. Also, the capacity is about 2750 vehicles per hour per lane (vphpl) if all vehicles are equipped. Overall or


Fig. 8. Capacity tendency.
on average, $R$ can represent the rate of market occupation of equipped vehicles. Consequently, eqn (32) indicates that if we seek to improve highway utilization with such an individual automatic vehicle-control system or intelligent cruise-control system, equipped vehicles must be popular. This takes time to accomplish. Perhaps, at the beginning of implementation of an automatic vehiclecontrol system like ADVANCE-F, announcing an extra traffic rule for equipped vehicles to keep left and non-equipped cars to keep right as rigidly as possible might be implemented in Taiwan. When the rate of market occupation of equipped vehicles exceeds half, planning an exclusive lane for equipped cars is a good alternative for traffic operation. The effectiveness of an Automated Highway System (AHS) has evidently been proven.
According to simulation results, ramp metering should be controlled to avoid vehicles queuing at the end of acceleration lanes, particularly during rush traffic; otherwise, the length of acceleration lanes should be increased.

## 5. CONCLUSION

As the number of passenger cars rapidly increases, traffic congestion becomes a continual problem in Taiwan. Extending highways is not a viable solution in urban areas. Construction costs are increasing rapidly. Public finance will not support it. Besides using a stricter policy for traffic management and control, developing high technology automatic driving modes to improve traffic problems is important. Project ADVANCE-F is an important subject in Taiwan. The fundamental traffic characteristics of mixed flows with equipped and non-equipped cars has been described. The effect on roadway capacity after implementing ADVANCE-F is as follows,

1. Implementing ADVANCE-F, an AVCS system, stabilizes the traffic stream.
2. Implementing ADVANCE-F enables a higher than conventional capacity limit of 2000 vehicles per hour per lane in ideal roadway conditions. The increment becomes more significant if the mixed ratio of equipped cars exceeds 0.5 . When all cars are equipped, the capacity increases about $33 \%$ to 2750 vphpl.

During the transition stage, it is necessary to propose suitable schemes of traffic control to achieve practical results. For instance, at the proper time, a lane separation policy with miscellaneous levels for equipped and non-equipped cars can be implemented. For a large rate of arrival of vehicles, ramp meters should be used to avoid queues occurring at the end of acceleration lanes. According to the simulation results, the effectiveness of an Automated Highway System (AHS) is proven. Based on a questionnaire in Taiwan, initially at least one third of car owners state an interest in this system (Chang, 1992). The traffic during the first decade of the twenty-first century will improve with the implementation of the ADVANCE-F system.

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